

Cosmic Flows: A Status Report

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Abstract. We give a brief review of recent developments in the study of the large-scale velocity field of galaxies since the international workshop on Cosmic Flows held in July 1999 in Victoria, B.C. Peculiar velocities (PVs) yield a tight and unique constraint on cosmological characteristics, independent of Λ and biasing, such as the cosmological matter density parameter (Ω_m) and the convergence of bulk flows on large scales. Significant progress towards incorporating non-linear dynamics and improvements of velocity field reconstruction techniques have led to a rigorous control of errors and much refined cosmic flow analyses. Current investigations favor low-amplitude ($\lesssim 250 \text{ km s}^{-1}$) bulk flows on the largest scales ($\lesssim 100h^{-1} \text{ Mpc}$) probed reliably by existing redshift-distance surveys, consistent with favored Λ CDM cosmogonies. Tidal field analyses also suggest that the Shapley Concentration (SC), located behind the Great Attractor (GA), might play an important dynamical role, even at the Local Group. Low-amplitude density fluctuations on very large scales generate the overall large-scale streaming motions while massive attractors like the GA, and Perseus-Pisces account for smaller scale motions which are superposed on the large-scale flow. Likelihood analyses of galaxy PVs, in the framework of flat CDM cosmology, now provide tight constraints of $\Omega_m = 0.35 \pm 0.05$. A four-fold size increase of our data base is expected in $\sim 4 - 5$ years with the completion of next generation FP/TF surveys and automated supernovae searches within $20,000 \text{ km s}^{-1}$.

1. Introduction

Ever since the discovery of the microwave background dipole by Smoot, Gorenstein & Muller (1977) and the pioneering measurements of galaxy motions by Rubin *et al.* (1976), the study of cosmic flows, or deviations from a smooth Hubble flow due to large-scale gravitational perturbations, has been recognized as one of the most powerful constraints to cosmological scenarios (Peebles 1980, Dekel 1994, Strauss & Willick 1995). Indeed, under the assumption that cosmic structure originated from small-amplitude density fluctuations that were

amplified by gravitational instability, the peculiar velocity \mathbf{v} and mass density contrast δ are together linked in the linear regime by a deceptively simple expression (from mass conservation in linear perturbation theory):

$$\nabla \cdot \mathbf{v} = -\Omega_m^{0.6} \delta. \quad (1)$$

The mean square bulk velocity on a scale R is easily calculated in Fourier space as:

$$\langle v^2(R) \rangle = \frac{\Omega_m^{1.2}}{2\pi^2} \int_0^\infty P(k) \widetilde{W}^2(kR) dk, \quad (2)$$

where $P(k)$ is the mass fluctuation power spectrum and $\widetilde{W}^2(kR)$ is the Fourier transform of a top-hat window of radius R . Measurements of galaxy PVs can thus directly constrain Ω_m , the shape and amplitude of the power spectrum, and test assumptions about the statistical properties of the initial fluctuations and gravitational instability as the engine of perturbation growth.

The last major workshop on Cosmic Flows in July 1999 in Victoria, BC (Courteau, Strauss, & Willick 2000; hereafter CFW2000) came at a time when important new data sets and critical modeling of the “biasing” relation between the galaxy and mass distribution were just being released. Fundamental questions debated at the conference, and central to all cosmological investigations based on cosmic flows, included¹: (1) *What is the amplitude of bulk flows on the largest scales probed?* (2) *Can velocity analysis provide accurate estimates of Ω_m ?*, and (3) *What is the value of Ω_m ?* The last two years have seen significant progress providing nearly definitive answers to each of the 3 questions above, as we discuss in the remainder of this review.

Detailed information about cosmic flows can be found in the *Cosmic Flows 1999* workshop proceedings (CFW2000), including the conference review by Dekel (2000). Also in Willick (1999) and Dekel (1999), as well as Willick (2000).

2. Data Sets and Bulk Flows

The radial peculiar velocity of a galaxy is derived by subtracting the Hubble velocity $H_0 d$ from the total velocity (redshift) cz in the desired frame of reference (*e.g.* CMB or Local Group). The distance d is inferred from a distance indicator (DI) whose accuracy dictates the range of applicability of the technique. The relative distance error of common DIs ranges from 20% (Tully-Fisher [TF], Fundamental Plane [FP], Brightest Cluster Galaxy [BCG]) down to 5-8% (Surface Brightness Fluctuations [SBF], SNIa, Kinetic Sunyaev-Zel’dovich [kSZ]). The bulk velocity \mathbf{V}_B of an ensemble of galaxies within a sphere (or a shell) of radius R is computed by a least square fit of a bulk velocity model predictions $\mathbf{V}_B \cdot \hat{\mathbf{n}}$ to the observed radial peculiar velocities, where $\hat{\mathbf{n}}$ is a unit vector in the direction of the object. Current results are summarized in Table 1 and represented graphically in Figure 1.

¹ Discussions about the measurements of the small-scale velocity dispersion and the coldness of the velocity field also figured prominently in the workshop agenda but we do not offer any update below, for lack of space. The interested reader should read CFW2000.

The data sets can be divided into two groups which lie either exactly within or somewhat above the predictions from most Λ CDM families. Fig. 1 shows the theoretical prediction of a Λ CDM model for the simplest statistic: the bulk-flow amplitude in a top-hat sphere. The solid line is the rms value, obtained by Eq. 2. The dashed lines represent 90% cosmic scatter in the Maxwellian distribution of V , when only one random sphere is sampled. With the exception of BCG, the directions of the non-zero flow vectors are similar (they all lie within 30° of $(l, b) = (280^\circ, 0^\circ)$) and the velocity amplitudes can be roughly compared even though the survey geometries and inherent sample biases can differ quite appreciably. A rigorous comparison of flow analyses must however account for different window functions (Kaiser 1988, Watkins & Feldman 1995, Hudson et al. 2000). Still, the obvious interpretation of these data is that of a gradual decline of the flow amplitude, or “convergence” of the flow field to the rest-frame of the CMB at $\sim 100h^{-1}$ Mpc, consistent with the theoretical assumption of large-scale homogeneity.

Cosmic variance however prevents any convergence to complete rest. Some of the reported error bars are based on a careful error analysis using mock catalogs, while others are crude estimates. In most cases they represent random errors only and underestimate the systematic biases. Large error bars for surveys such as BCG, LP10, SMAC, SNIa, and Shellflow, with fewer than a thousand “test particles,” are largely due to sampling errors which also increase with increasing volumes.

While present bulk flow estimates are in comforting agreement with current cosmologies, important efforts are currently underway to reduce the systematic and random errors inherent in most compilations of galaxy PVs, especially at large distance. The former is addressed by collecting homogeneous data across the entire sky, in the spirit of Lauer-Postman and Shellflow (Courteau et al. 2000). The latter simply requires that large numbers of galaxies and cluster of galaxies be observed to reduce Poisson noise and systematic biases. The nominal sample size to achieve a minimum signal/noise for each spherical volume chosen must be estimated from mock catalogs based on an expected number density profile (as a function of distance or redshift from us) and sky coverage. New surveys including many thousand “test” particles and reaching out to $15,000 \text{ km s}^{-1}$ should quantify the convergence of the peculiar velocity field on very large scales. These surveys include, for example, NFP² for the FP measurements of ~ 4000 early-type galaxies in 100 X-ray selected clusters, 6dF³ for the FP measurements of $\sim 15,000$ Southern hemisphere early-type galaxies, the SNfactory⁴ for the serendipitous detection and subsequent follow-up of a few hundred SNe per year (Aldering 2001, private communication), and the Warpfire⁵ extension of Lauer & Postman (1994)’s BCG analysis. These studies should be completed by 2005, if not sooner.

²astro.uwaterloo.ca/~mjhudson/nfp/

³msowww.anu.edu.au/colless/6dF/

⁴snfactory.lbl.gov. The detection range should actually extend out to $24,000 \text{ km s}^{-1}$.

⁵www.noao.edu/noao/staff/lauer/warpfire/

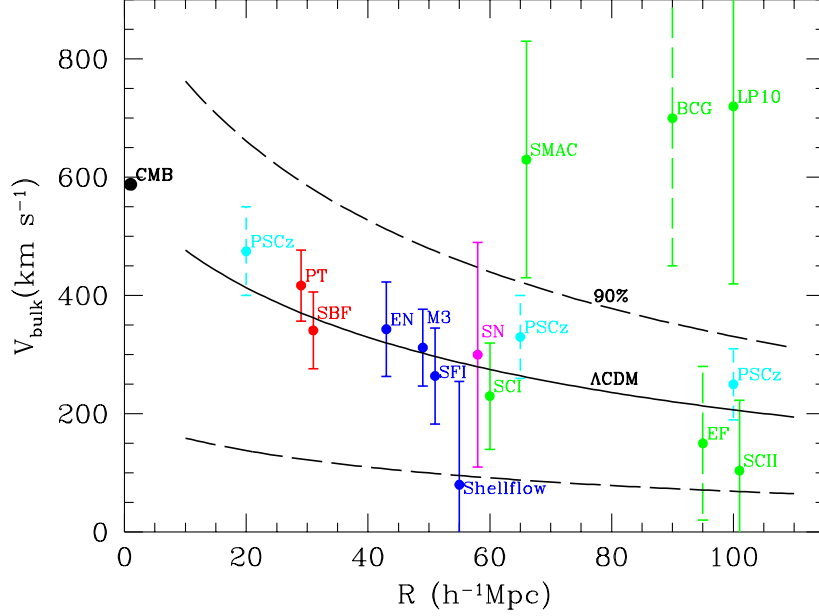


Figure 1. Amplitude of CMB bulk velocity in top-hat spheres about the LG, in comparison with theory. The curves are the predicted rms and cosmic scatter for a Λ CDM model. The measurements, based on the data listed in Table 1, are crudely translated to a top-hat bulk velocity. The error bars are random only. All the non-zero vectors (except BCG) point to $(l, b) = (280^\circ, 0^\circ) \pm 30^\circ$. Shown as well are the LG dipole velocity (labeled “CMB”), and linear estimates from the PSCz redshift survey for $\beta = 0.7$. Care must be exercised when interpreting such plots since directions are not plotted and projected amplitudes (V_X, V_Y, V_Z) may differ substantially (*e.g.* Hudson et al. 2000).

TABLE I. RECENT BULK FLOW MEASUREMENTS[†]

Survey	R_{eff} (km s ⁻¹)	V_B (km s ⁻¹)	Dist. Ind.
Lauer-Postman (BCG)	12500	700	BCG
Willick (LP10K)	11000	700	TF
Hudson et al. (SMAC)	8000	600	FP
Tonry et al. (SBF)	3000	290	SBF
Wegner et al. (ENEAR)	5500	340	D_n - σ
Dekel et al. (POTENT/M3)	6000	350	TF, D_n - σ
Riess et al. (SNIa)	6000	300	SN Ia
Courteau et al. (SHELLFLOW)	6000	70	TF
Dale & Giovanelli (SFI)	6500	200	TF
Colless et al. (EFAR)	10000	170	FP
Dale & Giovanelli (SCI/SCII)	14000	170	TF

[†] All references in CFW2000. With the exception of Lauer-Postman (1994), all results are post-1999.

2.1. The Large-Scale Tidal Field

The cosmological peculiar velocity field at any point can be decomposed into the sum of a “divergent” field due to density fluctuations inside the surveyed volume, and a tidal (shear) field, consisting of a bulk velocity and higher moments, due to the matter distribution outside the surveyed volume. This procedure was carried out by Hoffman et al. (2001), using reconstructions by POTENT (Dekel et al. 1999) or Wiener Filter (Zaroubi, Hoffman & Dekel 1999), with respect to a sphere of radius $60h^{-1}$ Mpc about the Local group. Their results are illustrated in Fig. 2. The divergent component is dominated by the flows into the Great Attractor (left) and Perseus-Pisces (right), and away from the void in between. The tidal field shows, for example, that about 50% of the velocity of the Local Group in the CMB frame is due to external density fluctuations. Their analysis suggests the non-negligible dynamical role of super-structures at distances of $100 - 200h^{-1}$ Mpc, specifically the Shapley Concentration and two great voids. These should be taken into account when considering the convergence of bulk velocity from different surveys on different scales and of the dipole motion of the Local Group.

3. Power Spectra and the Measurement of Ω_M

The peculiar velocities allow direct estimates of Ω_m independent of galaxy biasing and Λ . Early analyses have consistently yielded a lower bound of $\Omega_m > 0.3$ (*e.g.*, Dekel & Rees 1993), but not a tight upper bound.

Cosmological density estimates from the confrontation of PVs and the distribution of galaxies in redshift surveys have traditionally yielded values in the range $0.3 < \Omega_m < 1$ (95% confidence). This wide span has often been attributed to nontrivial features of the biasing scheme or details of the reconstruction/likelihood method such as the choice of smoothing length. Two common approaches to measuring Ω_m are known as the *density-density* (d-d) and *velocity-velocity* (v-v) comparisons. Density-density comparisons based on POTENT-like reconstructions (*e.g.*, Sigad et al. 1998) have produced typically large values of Ω_m , while v-v comparisons yield smaller estimates (*e.g.*, Willick et al. 1998 [VELMOD], Willick 2000, Nusser et al. 2001, Branchini et al. 2001). These differences have recently been shown to be insensitive to the complexity of the biasing scheme, whether it be non-linear, stochastic, or even non-local (Berlind et al. 2001; see also Feldman et al. 2001). Thus, one must look for differences inherent to d-d/v-v techniques for an explanation of their apparent disagreement.

Likelihood analyses of the individual PVs (*e.g.* Zaroubi et al. 1997, Freudling et al. 1999, Zehavi & Dekel 1999) can be used to estimate the power spectrum of density fluctuations under the assumption that these are drawn from a Gaussian random field. In linear theory, the shape of the power spectrum $P(k)$ does not change with time and thus provides a powerful tool to estimate basic cosmological parameters. Moreover, power spectrum analyses of PVs are free of the problems that plague similar determinations from redshift surveys such as redshift distortions, triple-valued zones, and galaxy biasing, and suffer from

weaker non-linear clustering effects. Likelihood methods simply require as prior a parametric functional form for $P(k)$.

The likelihood analysis of Silberman et al. (2001) incorporates a correction to the power spectrum for non-linear clustering effects, which has been carefully calibrated using new mock catalogs based on high-resolution simulations. The effect of this correction, shown in Fig. 3, is to account for larger power on small scales and suppress the overall amplitude of $P(k)$ on larger scales where clustering is still linear. An unbiased fit of $P(k)$ in the linear regime can thus be achieved, leading to unbiased constraints on the relevant cosmological parameters. The $P(k)$ prior in their analysis assumed a flat Λ CDM cosmological model ($h = 0.65, n = 1$, COBE normalized), with only Ω_m as a free parameter. Fig. 3 gives final fits based on the Mark III (Willick et al. 1997) and SFI (Haynes et al. 1999) catalogs of galaxy PVs. The Mark III catalog is more densely sampled at small distances than SFI and also includes elliptical galaxies which are absent in SFI; the correction for non-linear effects is thus stronger for Mark III. Fitted

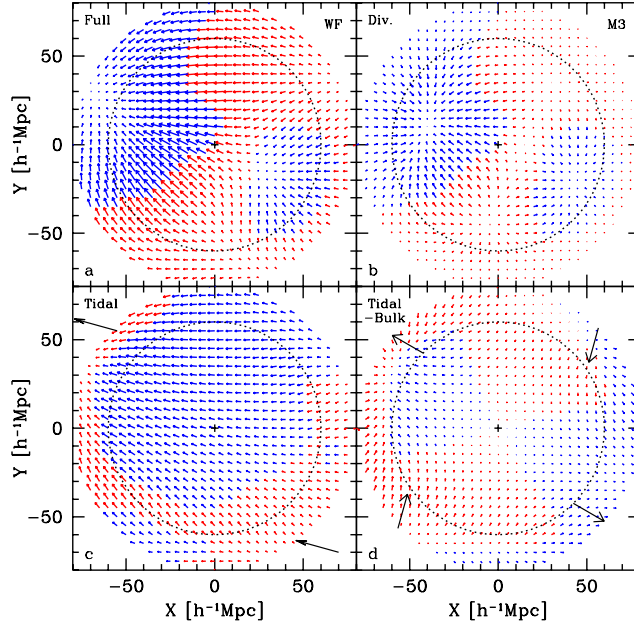


Figure 2. Wiener filter reconstruction by Hoffman et al. (2001) of the Mark III velocity field in the Supergalactic plane, with respect to the sphere of $60 h^{-1}\text{Mpc}$ about the Local Group (center). The velocities are measured in $h^{-1}\text{Mpc}$ ($1 h^{-1}\text{Mpc} = 100 \text{ km s}^{-1}$). (a) The full velocity field. (b) The divergent component due to the mass fluctuations within the sphere. (c) The tidal component due to the mass distribution outside the sphere. (d) The residual after subtracting the bulk velocity from the tidal component, including quadrupole and higher moments. The black long arrows in the bottom panels show the projected directions of the bulk velocity and two of the shear eigenvectors respectively. For more information, refer to Hoffman et al. (2001).

values for the Mark III data drop from $\Omega_m = 0.56 \pm 0.04$ in the earlier linear analysis to 0.32 ± 0.06 in the improved analysis, and for SFI from 0.51 ± 0.05 to 0.37 ± 0.09 . These revised tight constraints from PVs represent a significant improvement in this analysis.

These results are in broad agreement with a recent v-v likelihood analysis of SFI PVs against the PSCz IRAS redshift survey by Branchini et al. (2001).

Their procedure entails some assumptions about the biasing of IRAS galaxies for which PSC redshifts are measured. If linear biasing were invoked with a biasing parameter near unity, Branchini et al. would find even smaller values of the density parameter with $0.15 \leq \Omega_m \leq 0.30$. This exercise and a direct comparison with the PV-only likelihood analysis of, say, Silberman et al. is however futile without a proper prescription of galaxy biasing. The direct analysis of PVs by themselves has the advantage of being free of the complications introduced by galaxy biasing.

A χ^2 test applied by Silberman et al. to modes of a Principal Component Analysis (PCA) shows that the nonlinear procedure improves the goodness of fit and reduces a spatial gradient that was of concern in the purely linear analysis. The PCA allows to address spatial features of the data and to evaluate and fine-tune the theoretical and error models. It demonstrates in particular that the Λ CDM models used are appropriate for the cosmological parameter estimation performed. They also addressed the potential for optimal data compression using PCA, which is becoming important as the data sets are growing big.

Intriguingly, when Silberman et al. allow deviations from Λ CDM, they find an indication for a wiggle in the power spectrum: an excess near $k \sim 0.05 (h^{-1}\text{Mpc})^{-1}$ and a deficiency at $k \sim 0.1 (h^{-1}\text{Mpc})^{-1}$ — a “cold flow”. This may be related to a similar wiggle seen in the power spectrum from redshift surveys (Percival et al. 2001 [2dF]) and the second peak in the CMB anisotropy (*e.g.* Halverson et al 2001 [DASI]).

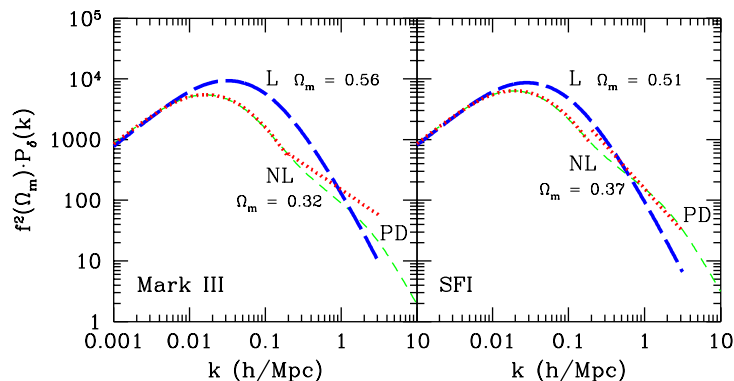


Figure 3. The recovered power spectra by the non-linear likelihood analysis of Silberman et al. (2001) from the data of M3 (left) and SFI (right). The $P(k)$ yielded by the purely linear analysis is marked “L”, while the nonlinear analysis, with a break at $k = 0.2 (h^{-1}\text{Mpc})^{-1}$, is marked “NL”. The corresponding values of Ω_m are marked. Also shown for comparison is an extrapolation of the linear part of the recovered $P(k)$ into the nonlinear regime by the Peacock-Dodds (1996) approximation. The $P(k)$ is in units of $(h^{-1}\text{Mpc})^3$.

4. The Future

Significant improvements in cosmic flow studies over the last couple of years include, for example: (1) unbiased recovery of cosmological parameters, such as Ω_m and $\sigma_8\Omega_m^{0.6}$, via quasi-nonlinear likelihood analyses of galaxy PVs; (2) modeling of non-linear clustering effects in power spectrum analyses from PVs, and implementing tools, based on PCA, for evaluating goodness of fit; and (3) better modeling of biased galaxy formation, in order to single out biasing in the comparison of PVs with redshift surveys and to generate proper mock catalogs for calibrating PV analysis methods.

Future developments rely heavily on growth of the available data bases and on refinements of existing catalogs. The VELMOD technique has enabled improved recalibrations of the Mark III (Willick et al. 1998) and SFI (Branchini et al. 2001) catalogs using external information from IRAS redshift surveys. We are planning an improved recalibration of Mark III using as backbone the homogeneous all-sky Shellflow sample, and merging all existing catalogs of PVs of field galaxies into a new Mark IV catalog.

A number of on-going and newly envisioned surveys (6dF, NFP, SNfactory, Warpfire) are expected to increase the size of existing data bases by a factor 4 within 2005. New wide-field surveys such as SLOAN, 2MASS, and DENIS will also provide most valuable complementary data to help control distance calibration errors.

A noticeable impact to precision flow studies should come from supernovae searches whose potential to build up very large catalogs of peculiar velocities (at the rate of a few hundred detections per year) and small relative error is unparalleled by no other distance indicator. (With $\Delta d/d(\text{SNIa}) \sim 8\%$, 1 SNIa is worth ~ 6 TF or FP measurements!) If a significant fraction of the new SNIe can be caught at peak light and monitored to measure a light curve (yielding precise distance estimates), current TF/FP data sets will be superseded in less than 5 years. Other ambitious surveys, such as those listed above, will complement accurate SN distances with very large data bases thus enabling remarkably tight flow solutions in the near future. There are good reasons to plan a new workshop on Cosmic Flows in 2005!

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We remain tremendously saddened by the departure of our friend and colleague Jeff Willick who did so much for the advancement of cosmic flow studies and who touched our lives very deeply.

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Piotr Popowski: A few years ago Avishai Dekel was measuring $\Omega_m = 1$ from cosmic flows based on his POTENT method, but now the value you quote is 0.3. Can you comment on what caused this difference?

Stéphane Courteau: Please note that all previous estimates of Ω_m reported by Dekel and others based on PVs *alone* (via POTENT or other methods) actually claimed a significant lower bound of $\Omega_m > 0.3$ but no tight upper bounds.

Their results at the time were consistent with $\Omega_m \sim 1$, but they never really claimed a measurement of $\Omega_m = 1$. The claimed lower bound is still valid, but now with the addition of a significant upper bound, ruling out $\Omega_m = 1$. The main improvement came from the incorporation of nonlinear effects in the likelihood analysis, which became possible due to proper mock catalogs based on high-resolution simulations.

A wider range of estimates has indeed been obtained by comparisons of PV data with galaxy redshift surveys. For example, a “d-d” comparison by Sigad et al. (1998) indicated a high value for Ω_m , while “v-v” comparisons, such as by VELMOD (Willick et al. 1998), yielded smaller values. These analyses were contaminated by galaxy biasing and nonlinear effects, which gave rise to relatively large uncertainties.